Section 2 – Equations and Inequalities

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.A-SSE.1.2	Use the structure of an expression to identify ways to rewrite it.
MAFS.912.A-REI.1.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MAFS.912.A-REI.2.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MAFS.912.A-CED.1.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational exponential functions.
MAFS.912.A-CED.1.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MAFS.912.A-CED.1.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MAFS.912.A-REI.4.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Topics in this Section

Section 2 – Topic 1
Equations: True or False?

Consider the statement $4 + 5 = 2 + 7$. This is a grammatically correct sentence.

Is the sentence true or false?

Consider the statement $1 + 3 = 8 + 6$. This statement is also a grammatically correct sentence.

Is the sentence true or false?

The previous statements are examples of ***number sentences***.

- A number sentence is a statement of equality between two ________________ expressions.
- A number sentence is said to be true if both numerical expressions are _________________.
- If both numerical expressions don't equal the same number, we say the number sentence is ___________.
- True and false statements are called ***truth values***.

Let's Practice!

1. Determine whether the following number sentences are true or false. Justify your answer.

   a. $13 + 4 = 7 + 11$

   b. $\frac{1}{2} + \frac{5}{8} = 1.4 - 0.275$

Try It!

2. Determine whether the following number sentences are true or false. Justify your answer.

   a. $(83 \cdot 401) \cdot 638 = 401 \cdot (638 \cdot 83)$

   b. $(6 + 4)^2 = 6^2 + 4^2$

A number sentence is an example of an ***algebraic equation***.

- An algebraic equation is a statement of equality between two ________________.
- Algebraic equations can be number sentences (when both expressions contain only numbers), but often they contain ______________ whose values have not been determined.

Consider the algebraic equation $4(x+2) = 4x + 8$.

Are the expressions on each side of the equal sign equivalent? Justify your answer.

What does this tell you about the numbers we can substitute for x?

Let's Practice!

3. Consider the algebraic equation $x + 3 = 9$.

   a. What value can we substitute for x to make it a true number sentence?

   b. How many values could we substitute for x and have a true number sentence?

4. Consider the algebraic equation $x + 6 = x + 9$. What values could we substitute for x to make it a true number sentence?

Try It!

5. Complete the following.

   a. $d^2 = 4$ is true for ______________________.

   b. $2m = m + m$ is true for ______________________.

   c. $d + 67 = d + 68$ is true for ______________________.

BEAT THE TEST!

1. Which of the following have the correct solution? Select all that apply.

- ☐ $2x + 5 = 19;\ x = 7$
- ☐ $3 + x + 2 - x = 16;\ x = 3$
- ☐ $\frac{x+2}{5} = 2;\ x = 8$
- ☐ $6 = 2x - 8;\ x = 7$
- ☐ $14 = \frac{1}{3}x + 5; x = 18$

Section 2 – Topic 2
Identifying Properties When Solving Equations

The following equations are equivalent. Describe the operation that occurred in the second equation.

$3 + 5 = 8$ and $3 + 5 - 5 = 8 - 5$

$x - 3 = 7$ and $x - 3 + 3 = 7 + 3$

$2(4) = 8$ and $\frac{2(4)}{2} = \frac{8}{2}$

$\frac{x}{2} = 3$ and $2 \cdot \frac{x}{2} = 2 \cdot 3$

This brings us to some more properties that we can use to write equivalent expressions.

Properties of Equality

If x is a solution to an equation, it will also be a solution to the new equation formed when the same number is added to or subtracted from each side of the original equation.

These are the ***addition and subtraction properties of equality***.

- If $a = b$, then $a + c = b + c$ and $a - c = b - c$.
- Give examples of this property.

If x is a solution to an equation, it will also be a solution to the new equation formed when the same number is multiplied by or divided into each side of the original equation.

These are the ***multiplication and division properties of equality***.

- If $a = b$, then $a \cdot c = b \cdot c$ and $\frac{a}{c} = \frac{b}{c}$.
- Give examples of this property.

Let's Practice!

1. The following equations are equivalent. Determine the property that was used to write the second equation.

   a. $x - 5 = 3x + 7$ and $x - 5 + 5 = 3x + 7 + 5$

   b. $x = 3x + 12$ and $x - 3x = 3x - 3x + 12$

   c. $-2x = 12$ and $\frac{-2x}{-2} = \frac{12}{-2}$

Try It!

2. The following equations are equivalent. Determine the property that was used to write the second equation.

   a. $2(x+4) = 14 - 6x$ and $2x + 8 = 14 - 6x$

   b. $2x + 8 = 14 - 6x$ and $2x + 8 + 6x = 14 - 6x + 6x$

   c. $2x + 8 + 6x = 14$ and $2x + 6x + 8 = 14$

   d. $8x + 8 = 14$ and $8x + 8 - 8 = 14 - 8$

   e. $8x = 6$ and $\frac{1}{8} \cdot 8x = \frac{1}{8} \cdot 6$

BEAT THE TEST!

1. For each algebraic equation, select the property or properties that could be used to solve it.

Algebraic Equation	Addition or Subtraction Property of Equality	Multiplication or Division Property of Equality	Distributive Property	Commutative Property
$\frac{x}{2} = 5$	○	○	○	○
$2x + 7 = 13$	○	○	○	○
$4x = 23$	○	○	○	○
$x - 3 = -4$	○	○	○	○
$4(x+5) = 40$	○	○	○	○
$10 + x = 79$	○	○	○	○
$-8 - x = 19$	○	○	○	○
$2(x-8) + 7x = 9$	○	○	○	○

Section 2 – Topic 3
Solving Equations

Sometimes, you will be required to justify the steps to solve an equation. The following equation is solved for x. Use the properties to justify the reasons for each step in the chart below.

Statements	Reasons
a. $5(x+3)-2=2-x+9$	a. Given
b. $5x+15-2=2-x+9$	b.
c. $5x+15-2=2+9-x$	c.
d. $5x+13=11-x$	d. Equivalent Equation
e. $5x+13-13=11-13-x$	e.
f. $5x=-2-x$	f. Equivalent Equation
g. $5x+x=-2-x+x$	g.
h. $6x=-2$	h. Equivalent Equation
i. $\frac{6x}{6}=\frac{-2}{6}$	i.
j. $x=-\frac{1}{3}$	j. Equivalent Equation

Other times, you may be required to write and solve an equation for a situation.

Consider the following scenario. Your class is raising funds for an end of the year trip to an amusement park. Your class plans to rent one bus. It costs $\$150.00$ to rent a school bus for the day plus $\$33.00$ per student admission ticket.

What is the variable in the situation?

Write an expression to represent the amount of money the school needs to raise.

Your class raised $\$1000$ for the trip. Write an equation to represent the number of students that can attend the trip.

Solve the equation to determine the number of students who can attend the trip.

Let's Practice!

1. Consider the following equation $2x - 3(2x - 1) = 3 - 4x$. Solve the equation for x. For each step, identify the property used to write an equivalent equation.

> **STUDY EDGE TIP**
>
> Some equations, such as $2x = 2x$ have **all real numbers** as the solution. No matter what number we substitute for x, the equation would still be true.

Try It!

2. Consider the following equation $3(4x + 1) = 3 + 12x - 5$. Solve the equation for x. For each step, identify the property used to convert the equation.

> **STUDY EDGE TIP**
>
> Some equations, such as $2x + 5 = 2x - 1$ have **no solution**. There is **NO** number that we could substitute for x that would make the equation true.

3. A high school surveyed its student population about their favorite sports. The 487 students who listed soccer as their favorite sport represented 17 fewer students than three times the number of students who listed basketball as their favorite sport. Write and solve an equation to determine how many students listed basketball as their favorite sport.

BEAT THE TEST!

1. The following equation is solved for x. Use the properties to justify the reasons for each step in the chart below.

Statements	Reasons
a. $2(x+5)-3=15$	a. Given
b. $2x+10-3=15$	b.
c. $2x+7=15$	c. Equivalent Equation
d. $2x+7-7=15-7$	d.
e. $2x=8$	e. Equivalent Equation
f. $\frac{2x}{2}=\frac{8}{2}$	f.
g. $x=4$	g. Equivalent Equation

Section 2 – Topic 4
Solving Equations Using the Zero Product Property

If someone told you that the product of two numbers is 10, what could you say about the two numbers?

If someone told you that the product of two numbers is zero, what could you say about the two numbers?

This is the ***zero product property***.

- If $ab = 0$, then either $a = 0$ or $b = 0$.

Describe how to use the zero product property to solve the equation $(x-3)(x+9)=0$. Then, identify the solutions.

Let's Practice!

1. Identify the solution(s) to $2x\ (x\ \ +4)(x\ \ +5)=0$.

2. Identify the solution(s) to $(2x\ \ -5)(x\ \ +11)=0$.

Try It!

3. Michael was given the equation $(x+7)(x-11)=0$ and asked to find the zeros. His solution set was $\{-11,7\}$. Explain whether you agree or disagree with Michael.

4. Identify the solution(s) to $2(y-3)\cdot 6(-y-3)=0$.

BEAT THE TEST!

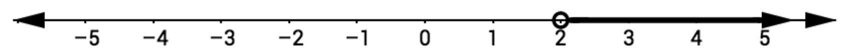
1. Use the values below to determine the solutions for each equation.

0	2	3	$\frac{4}{5}$
$\frac{2}{7}$	$-\frac{1}{2}$	$-\frac{3}{4}$	-14
6	0	$-\frac{1}{4}$	-2

$(2y+1)(y+14)=0$			
$(7n-2)(5n-4)=0$			
$(4x+3)(x-6)=0$			
$x(x+2)(x-3)=0$			
$t(4t+1)(t-2)=0$			

Section 2 – Topic 5
Solving Inequalities – Part 1

Let's start by reviewing how to graph inequalities.



- When the endpoint is an _________ dot or circle, the number represented by the endpoint _____ _________ a part of the solution set.

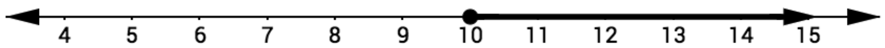
Describe the numbers that are graphed in the example above.

Can you list all the numbers? Explain your answer.

Write an inequality that represents the graph above.

Write the solution set that represents the graph above.

Consider the following graph.



- When the endpoint is a ____________ dot or circle, the number represented by the endpoint _______ a part of the solution set.

Write an inequality that represents the graph above.

Write the solution set that represents the graph above.

Why is "or equal to" included in the solution set?

Just like there are Properties of Equality, there are also ***Properties of Inequality***.

If $x > 5$, is $x + 1 > 5 + 1$? Substitute values for x to justify your answer.

Addition and Subtraction Property of Inequality

- If $a > b$, then $a + c > b + c$ and $a - c > b - c$ for any real number c.

Consider $(2x - 1) + 2 > x + 1$. Use the addition or subtraction property of inequality to solve for x.

Let's Practice!

1. Consider the inequality $(4 + x) - 5 \geq 10$. Use the addition or subtraction property of inequality to solve for x. Express the solution in set notation and graphically on a number line.

Try It!

2. Consider the following inequality $4x + 8 < 1 + (2x - 5)$. Use the addition or subtraction property of inequality to solve for x. Express the solution in set notation and graphically on a number line.

3. Peter deposited $\$27$ into his savings account, bringing the total to over $\$234$. Write and solve an inequality to represent the amount of money in Peter's account before the $\$27$ deposit.

Section 2 – Topic 6
Solving Inequalities – Part 2

Consider $x > 5$ and $2 \cdot x > 2 \cdot 5$. Identify a solution to the first inequality. Show that this solution also makes the second inequality true.

Consider $x > 5$ and $-2 \cdot x > -2 \cdot 5$. Identify a solution to the first inequality. Show that this solution makes the second inequality false.

How can we change the second inequality so that the solution makes it true?

Consider $-q > 5$. Use the addition and/or subtraction property of inequality to solve.

Multiplication Property of Inequality

- If $a > b$, then for any positive real number k, ak_____bk.
- If $a < b$, then for any positive real number k, ak_____bk.
- If $a > b$, then for any negative real number k, ak_____bk.
- If $a < b$, then for any negative real number k, ak_____bk.

The same property is true when dealing with ≤ or ≥.

Let's Practice!

1. Find the solution set to each inequality. Express the solution in set notation and graphically on a number line.

   a. $-9y + 4 < -7y - 2$

   b. $\frac{m}{3} + 8 \leq 9$

2. At 5:00 PM in Atlanta, Georgia, Ethan noticed the temperature outside was $72°\text{F}$. The temperature decreased at a steady rate of $2°\text{F}$ per hour. At what time was the temperature less than $64°\text{F}$?

Try It!

3. Find the solution set to the inequality. Express the solution in set notation and graphically on a number line.

   a. $-6\,(x-5) > 42$

   b. $4(x+3) \geq 2(2x-2)$

BEAT THE TEST!

1. Ulysses is spending his vacation in South Carolina. He rents a car and is offered two different payment options. He can either pay $\$25.00$ each day plus $\$0.15$ per mile (option A) or pay $\$10.00$ each day plus $\$0.40$ per mile (option B). Ulysses rents the car for one day.

   Part A: Write an inequality representing the number of miles where option A will be the cheaper plan.

   Part B: How many miles will Ulysses have to drive for option A to be the cheaper option?

2. Stephanie has just been given a new job in the sales department of Frontier Electric Authority. She has two salary options. She can either receive a fixed salary of $\$500.00$ per week or a salary of $\$200.00$ per week plus a 5% commission on her weekly sales. The variable s represents Stephanie's weekly sales. Which solution set represents the dollar amount of sales that she must generate in a week in order for the option with commission to be the better choice?

   Ⓐ $\{s|s > \$300.00\}$
   Ⓑ $\{s|s > \$700.00\}$
   Ⓒ $\{s|s > \$3{,}000.00\}$
   Ⓓ $\{s|s > \$6{,}000.00\}$

Section 2 – Topic 7
Solving Compound Inequalities

Consider the following options:

Option A: You get to play Call of Duty after you clean your room and do the dishes.

Option B: You get to play Call of Duty after you clean your room or do the dishes.

What is the difference in Option A and B?

Consider the following. Circle the statements that are true.

$2 + 9 = 11$ and $10 < 5 + 6$

$4 + 5 \neq 9$ and $2 + 3 > 0$

$0 > 4 - 6$ or $3 + 2 = 6$

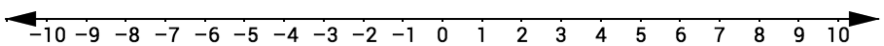
$15 - 20 > 0$ or $2.5 + 3.5 = 7$

These are called ***compound equations or inequalities***.

- When the two statements in the previous sentences were joined by the word *AND*, the compound equation or inequality is true only if __________ statements are true.

- When the two statements in the previous sentences were joined by the word *OR*, the compound equation or inequality is true if at least _________ of the statements is true. Therefore, it is also considered true if _________ statements are true.

Let's graph $x < 6$ and $x > 1$.



This is the __________________ ______________ to the compound inequality.

How many solutions does this inequality have?

Many times this is written as $1 < x < 6$. This notation denotes the conjunction "and."

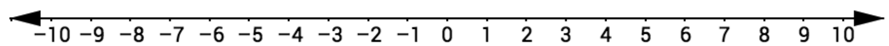
We read this as "x is greater than one ________ less than six."

Let's Practice!

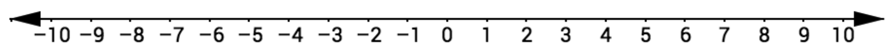
1. Consider $x < 1$ or $x > 6$. Could we write the inequalities above as $1 > x > 6$? Explain your answer.

2. Graph the solution set to each compound inequality on a number line.

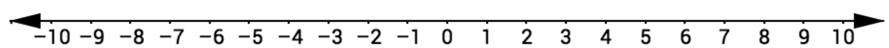
   a. $x = 2$ or $x > 5$



   b. $x > 6$ or $x < 6$



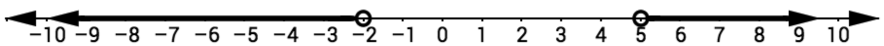
   c. $1 \leq -x \leq 7$



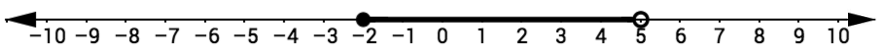
STUDY EDGE TIP

Be on the lookout for negative coefficients. When solving inequalities, you will need to reverse the inequality symbol when you multiply or divide by a negative value.

3. Write a compound inequality for the following graphs.



a. Compound inequality:

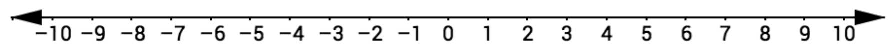


b. Compound inequality:

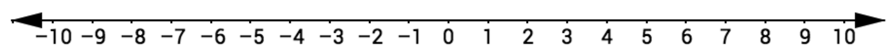
Try It!

4. Graph the solution set to each compound inequality on a number line.

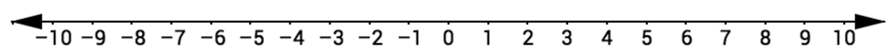
a. $x < 1$ or $x > 8$



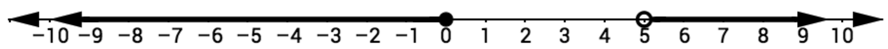
b. $x \geq 6$ or $x < 4$



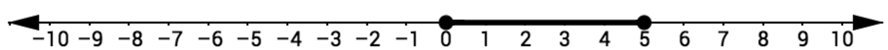
c. $-6 \leq x \leq 4$



5. Write a compound inequality for the following graphs.



a. Compound inequality:

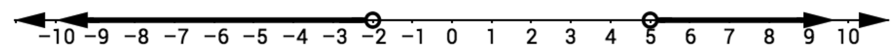


b. Compound inequality:

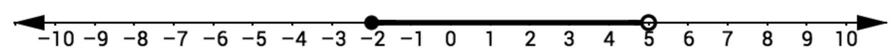
BEAT THE TEST!

1. Use the terms and symbols in the bank to write a compound inequality for each of the following graphs. You may only use each term once, but you do not have to use all of them.

$3x$	-14	-6	$\geq$	$-$	17	15	$<$
$7x$	$<$	2	or	$\leq$	$3x$	$+$	$>$



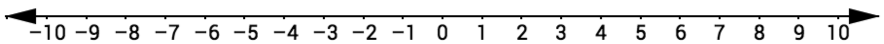
Compound Inequality:



Compound Inequality:

Section 2 – Topic 8
Solving Absolute Value Equations and Inequalities

Absolute value represents the distance of a number from zero on a number line.



How far away is "9" from zero on the number line?

This is written as ______________.

How far away is "−9" from zero on the number line?

This is written as ______________.

This is the ***absolute value*** of a number.

- For any real numbers c and d, if $|c| = d$, then $c = d$ or $c = -d$.

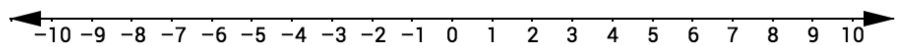
For example, $|f| = 5$, so $f =$ _____ or $f =$ _____.

Consider $|c| < 5$.

Using our definition of absolute value, this is saying that c represents all the numbers ____________ ____________ five units from zero on the number line.

What are some numbers that could be represented by c?

Graph all the numbers represented by c on a number line.



What is the solution set for c?

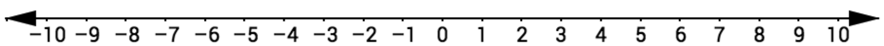
- For any real numbers c and d, if $|c| < d$ or if $|c| \leq d$, then $-d < c < d$ or $-d \leq c \leq d$.

Consider $|c| > 5$.

Using our definition of absolute value, this is saying that c represents all the numbers ___________ ________ five units from zero on the number line.

What are some numbers that could be represented by c?

Graph all the numbers represented by c on a number line.



What is the solution set for c?

- For any real numbers c and d, if $|c| > d$, then $c > d$ or $c < -d$.
- For any real numbers c and d, if $|c| \geq d$, then $c \geq d$ or $c \leq -d$.

Let's Practice!

1. Solve each absolute value inequality and graph the solution set.

   a. $|n + 5| < 7$

   b. $|a| + 3 > 9$

2. Tammy purchased a pH meter to measure the acidity of her freshwater aquarium. The ideal pH level for a freshwater aquarium is between 6.5 and 7.5 inclusive.

   a. Graph an inequality that represents the possible pH levels needed for Tammy's aquarium.

   b. Define the variable and write an absolute value inequality that represents the possible pH levels needed for Tammy's aquarium.

Try It!

3. Solve each equation or inequality and graph the solution set.

   a. $|p+7|=-13$

   b. $2|x|-4<14$

   c. $|2m+4|\geq 12$

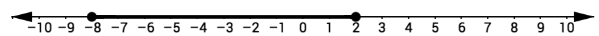
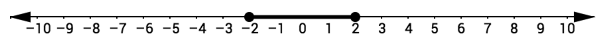
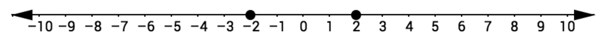
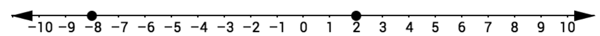
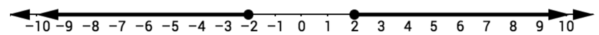
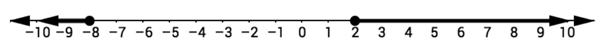
4. Baseball fans often leave a baseball game if their team is ahead or behind by five runs or more. Toronto Blue Jays fans follow this pattern, and the Blue Jays have scored eight runs in a particular game.

   a. Graph an inequality that represents the possible runs, r, scored by the opposing team if Toronto fans are leaving the game.

   b. Write an absolute value inequality that represents the possible runs, r, scored by the opposing team if Toronto fans are leaving the game.

BEAT THE TEST!

1. Match the following absolute value equations and inequalities to the graph that represents their solution.

		A. $\lvert x \rvert = 2$
		B. $\lvert x \rvert \geq 2$
		C. $\lvert x \rvert \leq 2$
		D. $\lvert x+3 \rvert \leq 5$
		E. $\lvert x+3 \rvert \geq 5$
		F. $\lvert x+3 \rvert = 5$

Section 2 – Topic 9
Rearranging Formulas

Solve each equation for x.

$2x + 4 = 12$

$2x + y = z$

Did we use different properties when we solved the two equations?

Consider the formula for the perimeter of a rectangle: $P = 2l + 2w$.

Sometimes, we might need the formula solved for length.

STUDY EDGE TIP

It is helpful to circle the variable that you are solving for.

Let's Practice!

1. Consider the equation $rx - sx + y = z$; solve for x.

Try It!

2. Consider the equation $8c + 6j = 5p$; solve for c.

3. Consider the equation $\frac{x-c}{2} = d$; solve for c.

BEAT THE TEST!

1. Isaiah planted a seedling in his garden and recorded its height every week. The equation shown can be used to estimate the height, h, of the seedling after w weeks since he planted the seedling.

$$h = \frac{3}{4}w + \frac{9}{4}$$

   Solve the formula for w, the number of weeks since he planted the seedling.

2. Shoe size and foot length for women are related by the formula $S = 3F - 24$, where S represents the shoe size and F represents the length of the foot in inches. Solve the formula for F.

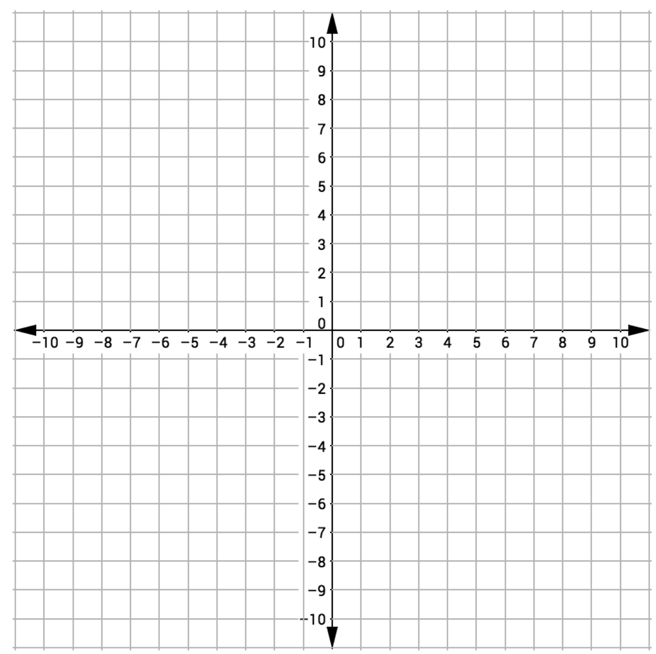
<u>Section 2 – Topic 10</u>
<u>Solution Sets to Equations with Two Variables</u>

Consider $x + 2 = 5$. What is the only possible value of x that makes the equation a true statement?

Now consider $x + y = 5$. What are some solutions for x and y that would make the equation true?

Possible solutions can be listed as ***ordered pairs***.

Graph each of the ordered pairs from the previous problem on the graph below.



What do you notice about the points you graphed?

How many solutions are there to the equation $x + y = 5$?

Let's Practice!

1. Tammy has 10 songs on her phone's playlist. The playlist features songs from her two favorite artists, Beyoncé and Pharrell.

   a. Create an equation using two variables to represent this situation.

   b. List at least three solutions to the equation that you created.

   c. Do we have infinitely many solutions to this equation? Why or why not?

STUDY EDGE TIP

In this case, our solutions must be natural numbers. Notice that the solutions follow a linear pattern. However, they do not form a line. This is called a ***discrete function***.

   d. Create a graph that represents the solution set to your equation.

   e. Why are there only positive values on this graph?

Try It!

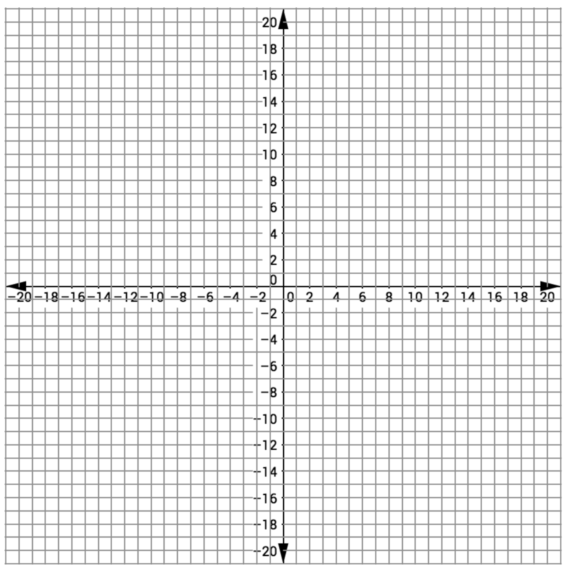
2. The sum of two numbers is 15.

    a. Create an equation using two variables to represent this situation.

    b. List at least three possible solutions.

    c. How many solutions are there to this equation?

    d. Create a visual representation of all the possible solutions on the graph.



STUDY EDGE TIP

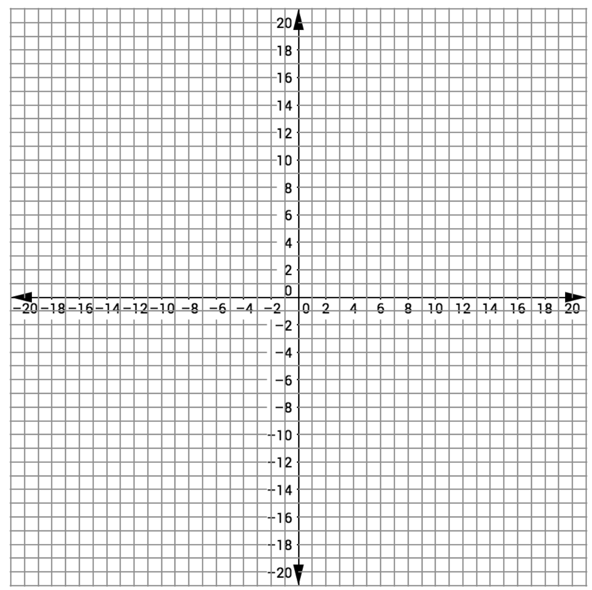
In this case, our solutions are rational numbers. Notice that the solutions form a line. This is called a ***continuous function***.

3. What if we changed the problem to say the sum of two integers is 15?

    a. Create an equation using two variables to represent this situation.

    b. Is this function discrete or continuous? Explain your answer.

    c. Represent the solution on the graph below.



BEAT THE TEST!

1. Elizabeth's tablet has a combined total of 20 apps and movies. Let x represent the number of apps and y represent the number of movies. Which of the following could represent the number of apps and movies on Elizabeth's tablet? Select all that apply.

- ☐ $x + y = 20$
- ☐ 7 apps and 14 movies
- ☐ $x - y = 20$
- ☐ $y = -x + 20$
- ☐ 8 apps and 12 movies
- ☐ $xy = 20$